of matter; and, therefore, all mathematical relations should have their analogue in the external world.

We have heretofore (see *Popular Science Monthly* for February 1874, p. 494) attempted to represent, by mathematical forms, the actual condition of the universe of matter, as here quoted: "There is, therefore, as I conceive, absolutely no limit to the division of matter, physically as well as mathematically; but our organization is such that, of the infinite series of terms in which it manifests itself, we can know, experimentally, only two; viz., the stellar universe, constituting the first *order*, of which the stars and the planets are the units; and, secondly, the chemical molecules, which constitute the second *order*.

"According to this view, the material universe might be represented in *orders* by the following series:

$$d^{-m}x, \ldots d^{-3}x, d^{-2}x, d^{-1}x, d^0x, dx, d^2x, d^3x, \ldots d^{n-1}x, d^nx,$$

in which x is the unknown quantity, called matter, and m and n are both infinitely great. In this series, d^0x, or simply x, would represent all tangible matter; and dx, which is the next term, *descending*, would represent the chemical molecule."

Whether the mathematical forms here presented are correct representations of the natural divisions of unorganized matter, or not, each individual will decide for himself, as no demonstration is, perhaps, possible; but that chemical combinations, and vegitable, and probably animal, organizations are represented in thought by algebraical forms, is, not only probable, but, a demonstrated fact; so that, in the language of Professor Sylvester, "Chemistry is the counterpart of a province of algebra, as probably the whole universe of fact is, or must be, of the universe of thought." (See *American Journal of Mathematics*, Vol. I, p. 83.)

TO FIND THE EARTH'S DISTANCE FROM THE SUN AT ANY GIVEN TIME.

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LET *AEP* represent the orbit of the Earth, and let S represent the Sun in one of the foci.

Let a and b be the semi axes of the orbit, E the position of the Earth n days from the perihelion, $r =$ the radius vector SE, $\varphi = PSE$, the angular distance of the Earth from the perihelion, and $c =$ the area daily passed over by the radius vector.

The polar equation of the orbit, if $e^2 = (a^2 - b^2) \div a^2$, is

$$r = \frac{a(1-e^2)}{1 + e \cos \varphi}. \tag{1}$$

The differential of a polar area is $\frac{1}{2} r^2 \varphi$;

$$\int \frac{a^2(1-e^2)^2 d\varphi}{2(1 + e \cos \varphi)^2} = nc;$$

or

$$\int \frac{d\varphi}{(1 + e \cos \varphi)^2} = \frac{2nc}{a^2(1-e^2)^2}.$$

To integrate $\dfrac{d\varphi}{(1 + e \cos \varphi)^2}$, assume $\cos \varphi = \dfrac{\cos \theta - e}{1 - e \cos \theta}$; $\qquad$ (2)

then

$$\frac{d\varphi}{(1 + e \cos \varphi)^2} = \frac{(1 - e \cos \theta) d\theta}{(1-e^2)^{\frac{3}{2}}}.$$

$$\int \frac{(1 - e \cos \theta) d\theta}{(1-e^2)^{\frac{3}{2}}} = \frac{\theta - e \sin \theta}{(1-e^2)^{\frac{3}{2}}} + C.$$

When $\varphi = 0$, $\theta = 0$ and $C = 0$; $\therefore$

$$\frac{\theta - e \sin \theta}{(1-e^2)^{\frac{3}{2}}} = \frac{2nc}{a^2(1-e^2)^2};$$

or

$$\theta - e \sin \theta = \frac{2nc}{a^2(1-e^2)^{\frac{1}{2}}}.$$

$$c = \frac{ab\pi}{T} = \frac{a^2 \pi \sqrt{(1-e^2)}}{T},$$

where T is the periodic time. $\therefore$

$$\theta - e \sin \theta = \frac{2n\pi}{T}.$$

But $\theta = \sin \theta + \frac{1}{6} \sin^3 \theta + \frac{3}{40} \sin^5 \theta + \frac{5}{112} \sin^7 \theta +$ &c., therefore by substitution and dividing by $1 - e$,

$$\sin \theta + \frac{1}{6(1-e)} \sin^3 \theta + \frac{3}{40(1-e)} \sin^5 \theta + \frac{5}{112(1-e)} \sin^7 \theta + \&c. = \frac{2n\pi}{(1-e)T}.$$

Reverting this series,

$$\sin \theta = \frac{2n\pi}{(1-e)T} - \frac{1}{6(1-e)} \left(\frac{2n\pi}{(1-e)T} \right)^3 + \frac{1+9e}{120(1-e)^2} \left(\frac{2n\pi}{(1-e)T} \right)^5$$

$$- \frac{1 + 54e + 225e^2}{5040(1-e)^3} \left(\frac{2n\pi}{(1-e)T} \right)^7 + \&c.$$

$\cos \theta = \sqrt{(1 - \sin^2 \theta)}$. Substituting in (2) we have $\cos \varphi$; and then substituting the value of $\cos \varphi$ in (1) we have r, the distance sought.